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Ketty PhDee

PART 8: *KETTIE* SHOOTING

Component metrics – Theoretical Band Physics

Forkers, it's time to graduate and go PHDee. In part 6, we explored Circular Error Probable (CEP / -or "circle of equal probability") as a measure of the *kettie's* precision in terms of USSP and calculated the Median Error Radius (MER), with applied GP–POI and GA–POI shot group shooting. In this part, we look at elastic band performance theory, some real-world flatband performance results, and discuss the significance of pouch metrics. You should think of your *kettie* as a propulsion system for the projectile required to impact the target, and - the engine is the flatbands.

Get the memo – it's PHdee time

We will deep dive into theory and focus on specific metrics (elements) of the PAM (Precision and Accuracy Matrix) as a practical resolve, to: "Our objective of this series is to get you as a *kettie* athlete – to the 10 m standard discipline shooting range line". We need to advance your (a) practical knowledge, (b) basic experience, and (c) critical equipment (such as "flatbands") to the next level. However, performance is still up to you – so keep an open mind and be ready for an expedient (d) elective knowledge learning curve (self-discovery) to be a competitive athlete, in one season. I will unequivocally not apologise for any incursion on industry prerogatives, the art-, cultural or historic beliefs regarding slingshot shooting. This is my teaching, use it – or lose it, period!

The legend

"Taper-cut bands shoot faster (yield more velocity) than straight-cut bands". We hear the statement frequently, and it has become a "resolute creed/rule" of conventional misguidance most athletes followed religiously (without questioning), with reasonable "perceived success" to further instil and fortify the allegory. I make the following statement regularly: "Basic rule is – the more aggressive the taper the more velocity (veracity of the contraction), thus the higher the speed, [but the lesser the torques (power) to move (set in motion) heavier pellets]". What if I were to say: "Elastic bands taper-cut' is grossly misunderstood and mostly incorrectly applied in practice, and that you should question the rationale, of when, why and how to use a taper-cut versus a straight-cut band? You see, we have to actually understand the fundamentals (underlying material and mechanical science), or the recipe (our winning plan) will always yield mediocre results (or- be significantly unreliable and inconsistent).

Précis

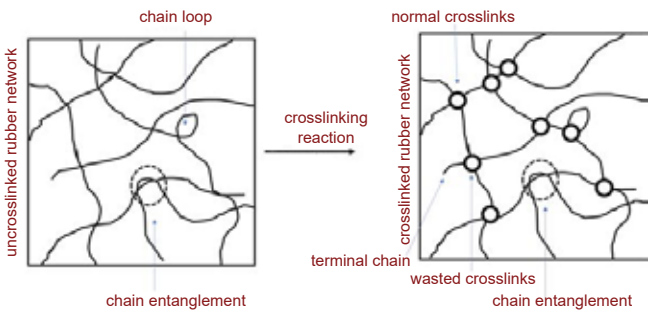
The basic hypotheses assumes that a "tapered band" will yield more velocity (give more speed) – but, less torques (power to set in motion, to propel) - as opposed to - a straight cut band [(the last, with the reverse effect) but, with the benefit of relative longevity]. Yes and No, the answer is not as straightforward as it seems, and perhaps more importantly for the slingshot athlete, it is rather important to understand the "tipping-point" of the performance yield model of the (i) Straight-cut (disadvantage-) to (ii) Tape-cut (advantage -) conversion.

Ketty PHDee Disclosure

The statement in question (taper-cut yield more velocity) - is (could be) true (valid), according to Ketty's Draw Weight Law: The velocity yield of a trapezoid (taper-cut) elastic band will exceed the rectangular (straight-cut) parent band, with the same draw weight (strength) at the same draw length. We will circle back to this very concerning revelation later. Note: The general major misperception and confusion revolve around – the assumption of draw length, misinterpreted to be – equal to draw weight. The draw weight for a set distance (example your draw length) change according to the geometrics of the band. Let us look at the basic – Slingshot Elastic (Flat-band)/ Rubber Physics Theory.

Theoretical (Slingshot) Elastic (Flatband) rubber physics

Your *kettie* flatband is broadly classified as an "elastomer" (elastic polymer), a viscoelastic polymer characterised with by the capacity to deform under stress and return to its original shape when the stress is released, a property commonly known as "elasticity". Viscoelasticity: Elastomers exhibit both elastic (spring-like) and viscous (fluid-like) behaviour, which means their response to stress is time-dependent. Polymer Structure: It is "long-chain polymers" in random three dimensions (3D) that allows for an extensive coiling intermingling network containing isoprene units for molecular structure cross-links. During the vulcanization manufacturing process with chemicals like dicumyl peroxide which decompose the long polyisoprene molecules chains and then chemically bonded (chemical nodes) at multiple points, resulting in a giant molecule (losing the information of the original long chain polymers. Thus a latex condom-, glove-, or your entire roll of flatband is one molecule. Wow! This network body's initial morphology, is governed by two random processes: (A) The isoprene units cross-link occurrences [statistical "probability"] - and the (B) random walk (flow) [reference Markov sequence] nature of the chain conformation (how it fills the mould space) and also responsible for the distance probability distribution for a fixed chain length (i.e. fixed number of isoprene units). In other words, it is the joining of isoprene units from one chain to another chain's isoprene unites units "probability distribution" of the network chain lengths and the end-to-end distances between their cross-link nodes that characterises the network morphology (the compound "characteristics"). The vulcanization temperature (example 150°C / even distribution, etc) and the actual mixing process of the base rubber and "cross-linked" agent chemicals (quality / quantity) such as, dicumyl or sulphur by design or accident (too much or inadequate), will affect the morphology (of the compound).



To really understand the elastic properties of your *kettie* bands; theoretically, it is necessary to know – both the physical mechanisms that occur at the molecular level and how the random-distribution nature of the polymer chain defines the 3D network, as defined (described) by the number of network nodes (isoprene connections) per unit volume and the statistical de-correlation length of the polymer. There should be three physical mechanisms immediately apparent, when stretching latex rubber bands – to produce the elastic forces within the network chains as the latex is stretched. Two (of the three-) forces arise from entropy changes, and the other one is associated with the distortion of the molecular bond angles along the chain backbone. Think of entropic forces in these polymer chains as a result of the thermal collisions that the constituent atoms encounter with the surrounding envi-

ronment. It is this constant ramming (collisions) of atoms that produces the elastic (resisting) force in the chains as they are forced (by the stress, as you pull) to become straight. A force constant for chain extension can be estimated from the resulting change in free energy associated with this entropy change. As a reference example a cross-link density of $4x10^{19}$ cm^{-3} an average chain will contain about 116 isoprene units (52 Kuhn lengths, at 2.2 isoprene units) and has a contour length of about 50 nm. Natural (latex) rubber (polyisoprene network), cross-linked with dicumyl peroxide, has tetra-functional cross-links, meaning each cross-link node has 4 network chains emanating from it. The isoprene unit has three single C-C (carbon) covalent [equilibrium state, chemical bond sharing electrons (shared pairs) in atoms] bonds, and there are two or three preferred rotational angles (orientations). The tensile force limit of the bonds can calculated via quantum chemistry simulations to an approximate value, example latex with 30 Mpa (30 $x106Nm^{-2}$) and a density of $0.92g/cm^{-3}$ and isoprene unit repeat molar mass (C_5H_8) 68.12g mol, approximation: 15 nN C-C bond [is equal to 15/1,000,000,000 /of a newton], about a factor of a thousand greater than the entropic chain forces. The angles between adjacent backbone C-C bonds in an isoprene unit vary between about 115 -120 degrees, and the forces associated with maintaining these angles are quite significant, so within each unit, the chain backbone always follows a zigzag path (even at bond rupture). This mechanism accounts for the steep upturn in the elastic stress, observed at high strains (taunt pull). Note: Caveat all, the 15 nN is under ideal conditions where the stress is equal in the entire network, and – it is a small number compared with the theoretical maximum conservative interatomic bond force (nN) scale and – the reason why a latex with 30 MPa breaks macroscopically long before it reached its Morse maximum.

At taut (high) stretch (optimised band tuning), some of the energy stored in the stretched network chain is due to a change in its entropy. However, most of the energy is stored in bond distortions, which do not involve an entropy change. As the latex rubber band is taut (stretched out), depending on the strain axis, and – in the case of a slingshot – single (linear?) directional tension (draw), each chain associated with an active cross-link node can have a different elastic force constant as it resists the applied strain (tension). To preserve force equilibrium (zero net force) on each cross-link node, a node may be forced to move in tandem with the chain having the highest force constant for chain extension. It is this complex node motion, arising from the random nature of the network morphology, that makes the study of the mechanical properties of rubber networks so difficult. As the network is strained, paths composed of these more extended chains emerge that span the entire band, and it is these paths that carry most of the stress at high strains. Because both the molecular physics mechanisms that produce the elastic forces and the complex morphology of the network (the elastic band) must be treated (calculated) simultaneously (inclusively), therefore simple "analytic elasticity models" are not possible (insufficient), as an explicit 3-dimensional numerical model is required to simulate the effects of strain on a representative volume element of the elastic band's network of chains.

Theoretical physics - the takeaway?

Although your rubber flatband is one molecule, it is very complex! There is no simple molecular or numerical model or methodology of study of latex flatband compounds simulation of latex rubber performance to predict the elasticity and durability, resilience, etc, as so often claimed and popularised by amateur non-scientific data bias exercises – to claim the "better or worse" of the other product or brand distinctions. Superior quality control of chemicals and processes is vital to produce slingshot bands fit for purpose for slingshot competition shooting, ensuring performance, reliability, consistency and durability. There is a lot more to the science of rubber physics, theories and models, such as material failure theory, yield point engineering (stress-strain curve), hysteresis, elasticity tensor, etc., but – let us touch on a few related metrics.

Snap-Back property (Restoring force)

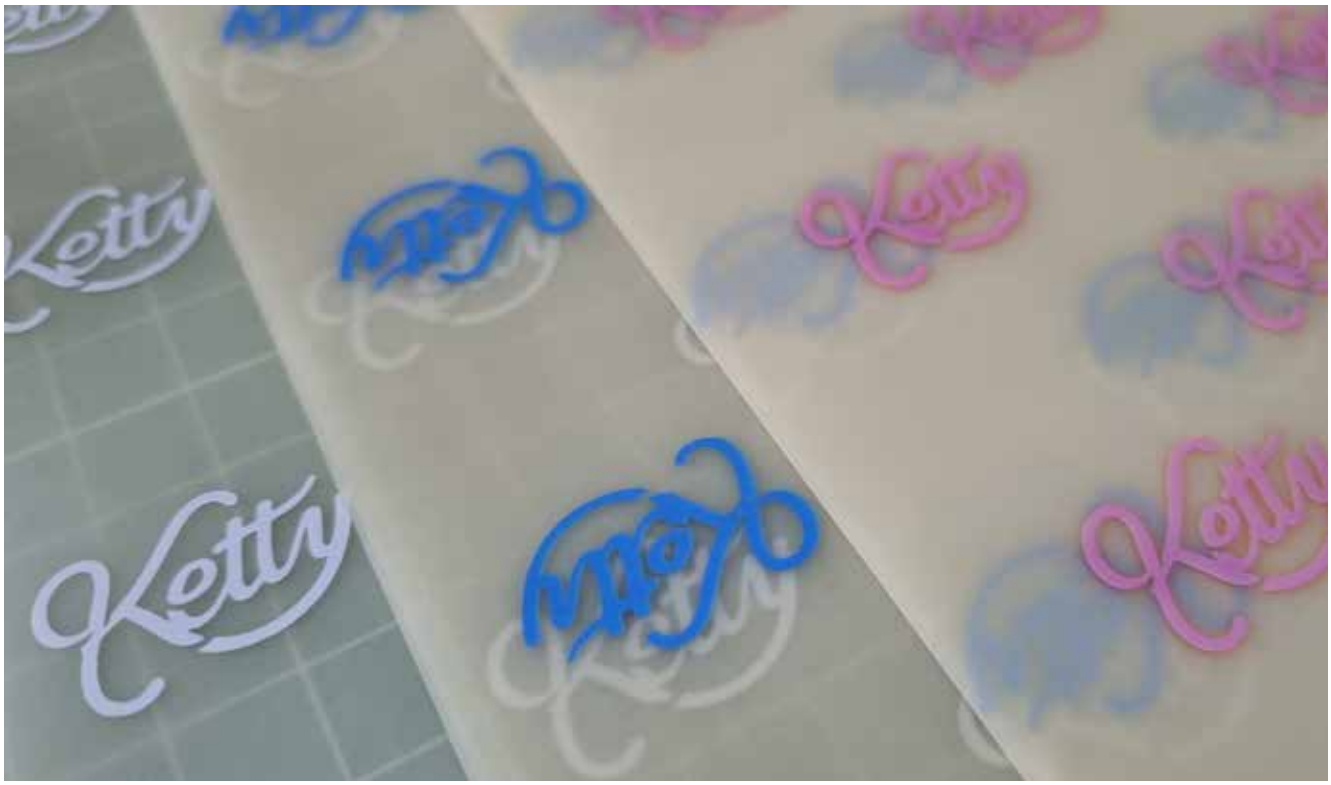
Depends on two factors, namely – the enthalpy and entropy change. When the pull (tension) force is applied, some rubber chains (in the network) are forced into a linearly orientation, resulting in a decrease in entropy in the latex rubber system, which gives rise to the elastic force in the network chains. Do you believe that your flatband contracts uniformly at once upon release?

The general flatband behaviour expectation

The belief is that when you pull (strain / stress) one end of the latex (rubber) elastic band (strip) and – release it, that it will (A) Snap back to its original size (length), in a (B) uniform (isotropic) contraction (across the entire span of the band (network). Similarly, to the implicit "stretching uniformity", if it stretches uniformly, it will contract uniformly. Right? **Ketty PHDee Disclosure:** The fundamental truth (fact) is quite the opposite: The contraction is anisotropic. In effect, the retraction process is deceptive, and- travels like a wave starting at the free (released) end. Although the observed "linear" behaviour of the displacement versus time suggests that, after an initial (free end) acceleration, both the free end and the midpoint of the band snaps back over time at a constant velocity. As you can imagine, – this plays directly into the matter of straight-cut versus taper-cut: Antics – or MASTERY! Equate into this revelation metrics like – the aerodynamic drag on the pouch, – and you have a marvellous conundrum.

The stress (draw force) and direction

The obvious mechanical stress (TCST) involved – is (A) Tension (draw) and (B) Compression (pull), but what about (C) Sheer (skew) at OTT angles, and – (D) Torsion (twist) of the



bands - in terms of (Di) pouch band tie and (Dii) pouch OTT or TTF position? There is a lot to consider, example: Euclidean Geometry affine (maps – of) transformation (or affinity) – geometric (automorphism) transformation (stretch) that maintains parallelism and lines preserving both the dimension of any affine subspaces (points to points, lines to lines, planes to planes) and – therefor the ratios of the lengths of parallel line segments, the distances between points lying on a straight line (not necessarily Euclidean spaces (distances / angles). Ponder – the question, I posed in the part 1 of the Ketty PHDee series: (1) Why is OTT more accurate but offers less precise (precision) versus TTF (more precision but less accurate)? What are the metrics involved and the scientifically measurable (calculable) difference that gives TTF better statistical precision? Top Tip: The draw on OTT bands is not uniform (isotropic), it is asymmetric (lopsided) with sheer forces at work.

Performance formula

In the effort to define latex rubber performance, – most "gurus" resort to **Hooke's law**: $F=kx$, where F is the force, k is the spring constant, and x is the distance. (The neo-Hookean model (1947) can be used on cross-linked polymers to predict their stress-strain relations). The empirical Robert Hooke's law 1678 *ut tensio, sic vis* ("as the extension, so the force" or "the extension is proportional to the force"). It states that the force (F) needed to extend or compress a spring by some distance (x) scales linearly with respect to that distance – that is, $F_s = kx$, where k is a constant factor characteristic of the spring (i.e., its stiffness), and x is small compared to the total possible deformation of the spring. It should be CLEARLY noted – the law is only a first order "linear approximation" for ideal conditions (equal / linear stress), rather than the real response of elastic bodies to applied forces. Taper-cut bands' sheer stress is obvious, but – does straight-cut elastic band material really extend ideal equally? Note: Many materials (such as latex rubber) will noticeably deviate from Hooke's law well before the elastic limits are reached, exactly the datum where slingshot athletes optimise ("tune") their bands. We will circle back to the Hook law in good time.

Straight-Cut Flatbands – characteristics

Width: Constant from fork to pouch. Shape: Rectangular strip. Cross-sectional area: Uniform along the length of the band. Force distribution: Even tension along the entire band. **Force vs. Draw Characteristics:** Uniform cross-sectional area, thus a "constant effective", and importantly – produces a linear draw force curve.

Taper-Cut Flatbands – characteristics

Width: Wider at the fork end and narrower at the pouch end. **Shape:** Trapezoidal (wider base tapering down). **Cross-sectional area:** Decreases along the band. **Force distribution:** Higher strain and energy concentration near the pouch end. **Force vs. Draw Characteristics:** Varying cross-sectional area, thus variable. **More efficient:** Lighter near the pouch = less mass to accelerate = better energy transfer. Non-linear draw force curve, and- often smoother and steeper toward full draw.

Question 1: Straight-cut bands vs. taper-cut bands?

Which performs best for -COMPETITION purposes? Read the question carefully, for COMPETITION purposes.

Reference point

Remember the objective, “fit for purpose”! What is required to successfully knock down targets in SASF (South African Slingshot Federation) standard indoor disciplines consistently? SASF (South African Slingshot Federation) rules, standardized on 8 mm steel balls, with a mass not less than 30 grains (1.94 grams) and not more than 35 grains (2.26 grams) – to knock down certified 2 fp/e (foot pound energy) targets, which translates to: At an average pellet size and mass of 32.6 grains (2.11 grams) at minimum velocity (speed) of 200 fs (feet per second) generating 3 fp/e, rather ideally you want 220 to 240 fs (<4 fp/e), depending on your draw.

It is much like “firearms ballistics - thinking”. You do not buy or build a small arms platform (SAP / firearm) and then try to find a bullet to work in that rifle! You rather select a bullet design (type, with your objective in mind (fit for that purpose (terminal velocity, etc) and determine the required (bullet mass, spin, etc) to project (propel, to target) the bullet, example a 100 meter bench rest, 1000 meter Bisley, hunting - pigs or elephant, etc.) Then, –when you understand and calculated the ballistics – you purposefully build the SAP to deliver the bullet as required to the target. Think of your slingshot platform (rig) in the same way. You need to deliver a pellet to knock down a target that requires a 2 fp/e (foot pound energy) impact to terminate.

Answer

In order to answer the question – it is time to ruffle some flatbands with scientific empirical (experiential) PHDee laboratory and field trial comparisons. Let’s do some – “bands to bands” assessment, with the limitations: Band gauges (thickness) from (A) the same gauges, (B) same production batches, (C) same pouches (and tie methods), (D) same steel pellets, (E) consistent environment (temperature and wind) – with a proper test bench set-up, to ensure (F) consistent angles, draw lengths, etc, and (G) release unbiased, with a (H) valid statistical data set of 30 shots each, (H) at a modest draw length of 67 cm, to accommodate the large youth demographic (I) band stretch is weak to moderate 4.2 ratio (instead of optimized / suggested 5 x) relaxed length (to accommodate poor band tuning). Notes: Velocity was (should be) measured 30 cm from the fork (bow), each band was fitted and warmed up with three (3) draws at 1.0 meters p/s (per second), same with each of the data point shot draws, 20 second delay between shot releases, with shot release in 2 seconds on dead-stop draw. The draw weight measured with certified push-pull instrumentation. Bow (fork) – model: Ketty Phantom, prongs set at 90 mm width, OTT (Over the Top) with fork (bow) at 180 degrees (upright).

I would like to invite you to participate in this scientific field trial – to establish some baseline / benchmark information: Work with what you have, record your findings.

Which performs best for - COMPETITION purposes?				
KETTY PROFESSIONAL ELITE - Flatbands Taper Cut vs. Straight Cut				
Pro Elite	Pellet mm	Draw (Pull) length: Temp:	Metrics Ve (Ave)	 Draw weight (Ave)
0.45 YELLO	8	15mm x 190		
0.50 Pink	8	20 mm x		
0.50 Pink	8	20mm-12mm x 190		
The MR / CEP pellet drop:				
0.55 Blue	8	20 mm x		
0.55 Blue	8	20mm-12mm x 190		
The MR / CEP pellet drop:				
0.60 White	8	20 mm x		
0.60 White	8	20mm-12mm x 190		
The MR / CEP pellet drop:				
Commercial Band Set:				

Record your: Observations: ? Deductions: ? Summation: ?

In the next part (9) you can compare notes on our findings. Good luck, Safe Shooting!

Thank you for joining me and participating in this sling-shot discovery journey. In the next part we will revisit Hook's Law and a few formulas, ask more questions and continue our real world flatband performance shooting and discuss the significance of pouch metrics and circle back to pellet jump – leading on to shooting actual shooting techniques.